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Stability of a Dual-Spin Satellite with a Four-Mass Nutation Damper

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The purpose of this Note is to suggest a four-mass nutation damper for use in dual-spin satellites. It is shown that the stability criteria developed for this damper do not involve the kind of design constraints which restricted the performance of the other available nutation dampers.[1–3] The result is found to hold even when a circular disk or wheel of uniform mass distribution is used to replace the four-mass structure in the proposed design.

Dual-Spin System

Consider the dual-spin configuration illustrated in Fig. 1. The main body of the satellite is a right circular cylinder with the nominal spin axis in the a_3 direction and a small rotor or fly-wheel is attached to it whose spin axis is also assumed to be along the a_3 body axis. The nutation damper placed only on the main body of the satellite, is a four-mass structure constrained to move in the a_2-a_3 plane as shown. Let 0 represent the center of mass of the whole system, and since the damper masses are arranged as collinear pairs, it can be assumed that there will be no appreciable shift in the center of mass of the system as the damper masses execute motions. It is also assumed that the four-mass structure is pivoted on a torsional system which offers dissipative torque in addition to the restoring torque. Although the results derived here will be found valid when a relatively more attractive structure—a wheel of uniform mass distribution—is used to replace the four-mass design shown in Fig. 1, the present analysis will be carried out for this latter design, since the four-mass configuration may be shown to be the simplest elementary structure which yields the desired results.

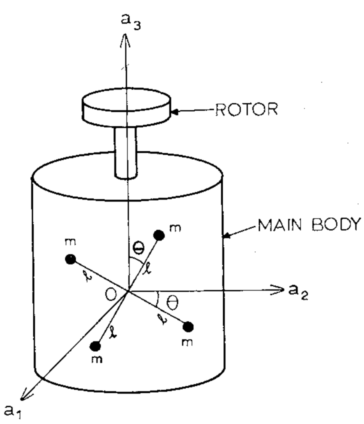

Fig. 1 The dual-spin system.

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Equations of Motion

For the system of Fig. 1, the equation of motion in a torque-free environment may be written as[2]

$$\pi'\cdot\boldsymbol{\omega} + \pi\cdot\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times(\pi\cdot\boldsymbol{\omega}) + \dot{\mathbf{h}} + m\sum_{i=1}^{4}\mathbf{r}_i\times\mathbf{r}''_i + m\sum_{i=1}^{4}\boldsymbol{\omega}\times(\mathbf{r}_i\times\mathbf{r}'_i) = 0 \quad (1)$$

where π represents the inertia dyadic of the total system about the point 0, $\boldsymbol{\omega}$ is the angular velocity of the main body of the satellite in inertial space, $\mathbf{r}_i$ is the vector from 0 to the damper masses m, and $\mathbf{h}$ is the relative angular momentum of the rotor and where the prime over a quantity indicates the time derivative in the reference body frame, while the dot indicates inertial differentiation.

First, in evaluating the terms in Eq. (1) involving the inertia dyadics, note that the effective moments and products of inertia for the system of Fig. 1 are given by

$$I_{11} = I_1, I_{22} = I_2, I_{33} = I_3$$

and

$$I_{12} = I_{21} = I_{13} = I_{31} = I_{23} = I_{32} = 0 \quad (2)$$

where I_1, I_2, and I_3 represent the moments of inertia of the dual-spin system under consideration when the damper masses are not deflected from their initial positions on the body axes.

Also, by definition,

$$\pi = \Sigma I_{\alpha\beta}a_\alpha a_\beta \quad (3)$$

Therefore, by using Eqs. (2) and (3), the terms involving the inertia dyadics in Eq. (1) may be evaluated as

$$\pi'\cdot\boldsymbol{\omega} + \pi\cdot\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}\times(\pi\cdot\boldsymbol{\omega}) = a_1[I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2)] + a_2[I_2\dot{\omega}_2 - \omega_1\omega_3(I_3 - I_1)] + a_3[I_3\dot{\omega}_3 - \omega_1\omega_2(I_1 - I_2)] \quad (4)$$

Next, the term $\dot{\mathbf{h}}$ which represents the contribution from the rotor to the basic equation of motion, may be found in the component form as

$$\dot{\mathbf{h}} = a_1I_R\omega_2\Omega - a_2I_R\omega_1\Omega + a_3I_R\dot{\Omega} \quad (5)$$

where I_R denotes the spin-axis moment of inertia of the rotor and Ω is the angular speed of the rotor relative to the main body of the satellite.

Finally, by expressing the vector quantity $\mathbf{r}_i$ in terms of its a_2 and a_3 components, it can be shown that the evaluation of the two summation terms in Eq. (1) yields

$$m\sum_{i=1}^{4}\mathbf{r}_i\times\mathbf{r}''_i + m\sum_{i=1}^{4}\boldsymbol{\omega}\times(\mathbf{r}_i\times\mathbf{r}'_i) = -a_1I_m\ddot{\theta} - a_2I_m\omega_3\dot{\theta} + a_3I_m\omega_2\dot{\theta} \quad (6)$$

where $I_m = 4ml^2$ is the moment of inertia of the damper mass structure.

Now, by combining Eqs. (4, 5, and 6), one obtains, under the assumed torque-free condition, the following set of nonlinear equations:

$$I_1\dot{\omega}_1 - \omega_2\omega_3(I_2 - I_3) + I_R\omega_2\Omega - I_m\ddot{\theta} = 0 \quad (7)$$

$$I_2\dot{\omega}_2 - \omega_1\omega_3(I_3 - I_1) - I_R\omega_1\Omega - I_m\omega_3\dot{\theta} = 0 \quad (8)$$

$$I_3\dot{\omega}_3 - \omega_1\omega_2(I_1 - I_2) + I_R\dot{\Omega} + I_m\omega_2\dot{\theta} = 0 \quad (9)$$

These equations involve five unknowns, viz., ω_1, ω_2, ω_3, θ, and Ω and, therefore, two additional equations are required to form a complete set. Clearly, one is provided by the equation of rotor motion, given by

$$I_R(\dot{\omega}_3 + \dot{\Omega}) = T \quad (10)$$

where T represents the magnitude of the moment applied about the rotor axis, while the other equation is obtained by considering the rotational motion of the damping system caused by the inertial forces. Thus, by computing the moments about the a_1 axis due to both the a_2 and a_3 components of inertial forces acting on all the four masses of the nutation damper considered, and noting that the resulting torque simply equals the sum of the damping torque and the restoring torque offered by the torsional spring arrangement about the point 0, this latter equation may be found as

$$I_m\ddot{\theta} - I_m\dot{\omega}_1 + C\dot{\theta} + K\theta = 0 \quad (11)$$

where C denotes the damping rate constant and K is the spring constant of the proposed damper.

Stability Analysis

The Eqs. (7–11) describe completely the behavior of the dual-spin system of Fig. 1 with the proposed four-mass nutation damper. It may be seen that all these equations are trivially satisfied for the solution

$$\omega_3 = \text{const}$$
$$\Omega = \text{const}$$

and

$$\omega_1 = \omega_2 = \theta = 0 \quad (12)$$

provided that $T = 0$, meaning thereby that the motor torque balances the bearing friction. Stability of this solution can be studied by considering the following set of linear equations:

$$\dot{\omega}_1 + \lambda\omega_2 - I_{m_1}\ddot{\theta} = 0 \quad (13)$$

$$\dot{\omega}_2 - \lambda\omega_1 - I_{m_1}\omega_3\dot{\theta} = 0 \quad (14)$$

and

$$-\dot{\omega}_1 + \ddot{\theta} + \beta\dot{\theta} + p^2\theta = 0 \quad (15)$$

where the variables now represent small increments around the stable solution given by Eq. (12) and where the constants are defined as follows:

$$\lambda = [(I_3 - I_T)\omega_3 + I_R\Omega]/I_T \quad (16)$$

$$I_{m_1} = I_m/I_T \quad (17)$$

where, due to the symmetry, it is assumed that $I_1 = I_2 = I_T$, I_T being the transverse moment of inertia of the total system and, additionally, $\beta = C/I_m$ and $p^2 = K/I_m$.

By using Eqs. (13–15), the characteristic equation may be obtained as

$$(1 - I_{m1})s^4 + \beta s^3 + (p^2 + \lambda^2 + I_{m_1}\lambda\omega_3)s^2 + \lambda^2\beta s + \lambda^2p^2 = 0 \quad (18)$$

This is a fourth-order polynomial with constant coefficients. The application of Routh-Hurwitz criterion to this polynomial will give both the necessary and sufficient conditions for asymptotic stability of the solution expressed by Eq. (12). Thus, for stability, the procedure yields

a) $(1 - I_{m_1}) > 0$

b) $\beta > 0$

c) $(p^2 + I_{m_1}\lambda\omega_3 + \lambda^2 I_{m1}) > 0$

d) $I_m\beta\lambda^3(\omega_3 + \lambda) > 0$

and

e) $\lambda^2p^2 > 0$ (19)

Note that the stability conditions given do not involve the kind of design constraints which restricted the performance of the other available nutation dampers. From Eq. (19), the following stability criteria may finally be derived:

a) $\lambda > 0$

b) $\beta > 0$ (20)

Use of a Circular Disk or Wheel

A logical extension of the four-mass nutation damper described will be the case where a circular disk or wheel of uniform mass distribution is used to experience the torque produced by the inertial forces. It can be shown that the same stability criteria given by Eq. (20), will also hold in this situation with the only difference that I_m will now represent the moment of inertia of the wheel used.

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Comparison of Beam Impact Models

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Nomenclature

A = cross-sectional area
A_s = cross-sectional area contributing to resistance to shear
E = modulus of elasticity
G = modulus of rigidity
I = cross-sectional moment of inertia
L = span length
s = spring stiffness
t = time
x = space variable measured along beam axis
y = vertical deflection of neutral axis
ψ = rotation of cross section about neutral axis

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